Instructions:

- This exam contains 12 pages. When we begin, check you have *one* of each page.
- You will have 75 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.**
  In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:

By writing my name below, I agree that all the work which appears on this exam is entirely my own.

I will not look at other peoples' work, and I will not communicate with anyone else about the exam.

I will not use any calculators, notes, etc.

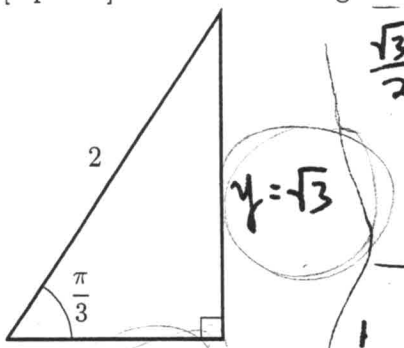
I understand that violating the above carries *serious consequences*, both moral and academic.

Printed Name: Key Signature: _____

Section: _____

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	12	8	8	12	12	8	8	10	12	10	100
Score:											

1. (a) [6 points] Fill in the missing sides of the triangle.



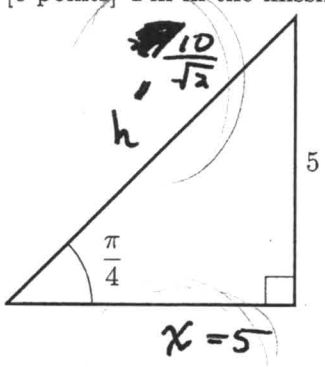
$\frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{3}\right) = \frac{y}{2}$

$\Rightarrow \quad y = \sqrt{3}$

$\frac{1}{2} = \cos\left(\frac{\pi}{3}\right) = \frac{x}{2}$

$\Rightarrow x = 1$

(b) [6 points] Fill in the missing sides of the triangle.



$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \frac{5}{h}$

$\sqrt{2} \cdot h = 10$

$h = \frac{10}{\sqrt{2}}$

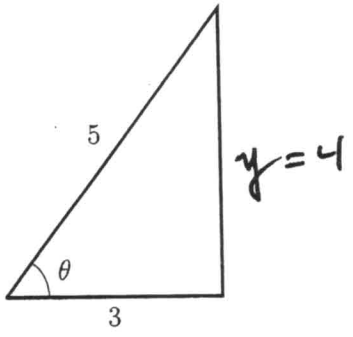
$x^2 + 5^2 = h^2$

$x^2 + 25 = \frac{100}{2} = 50$

$x^2 = 25$

$x = 5$

2. [8 points] Compute all 6 trig functions using this triangle.



$y^2 + 3^2 = 5^2$

$y^2 + 9 = 25$

$y^2 = 16$

$y = 4$

$\sin(\theta) = \frac{opp}{hyp} = \frac{4}{5}$ (2)

$\cos\theta = \frac{adj}{hyp} = \frac{3}{5}$ (2)

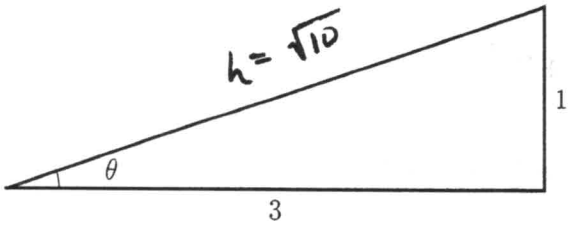
$\tan\theta = \frac{opp}{adj} = \frac{4}{3}$ (1)

$\csc(\theta) = \frac{1}{\sin\theta} = \frac{5}{4}$ (1)

$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}$ (1)

$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$ (1)

3. [8 points] Compute all 6 trig functions using this triangle.



$1^2+3^2=h^2$

$1+9=h^2$

$h=\sqrt{10}$

$\sin\theta = \frac{opp}{hyp} = \frac{1}{\sqrt{10}}$ 2

$\cos\theta = \frac{adj}{hyp} = \frac{3}{\sqrt{10}}$ 2

$\tan\theta = \frac{opp}{adj} = \frac{1}{3}$ 1

$\csc(\theta) = \frac{1}{\sin\theta} = \sqrt{10}$ 1

$\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{10}}{3}$ 1

$\cot\theta = \frac{1}{\tan\theta} = 3$ 1

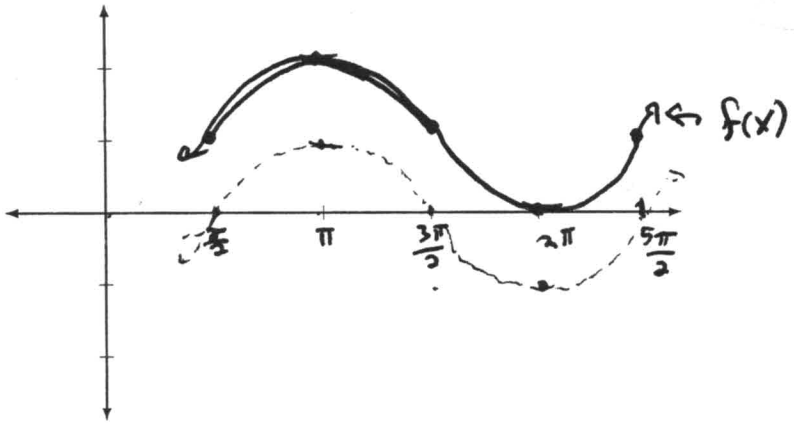
6:57

4. Graph modifications of Sine and Cosine, and give their period.

(a) [6 points] $f(x) = \sin(x - \frac{\pi}{2}) + 1$

Be sure you fill in the scale for the x and y axes.

1 sin(x) 2
moved right $\frac{\pi}{2}$ 2
and up 1 2

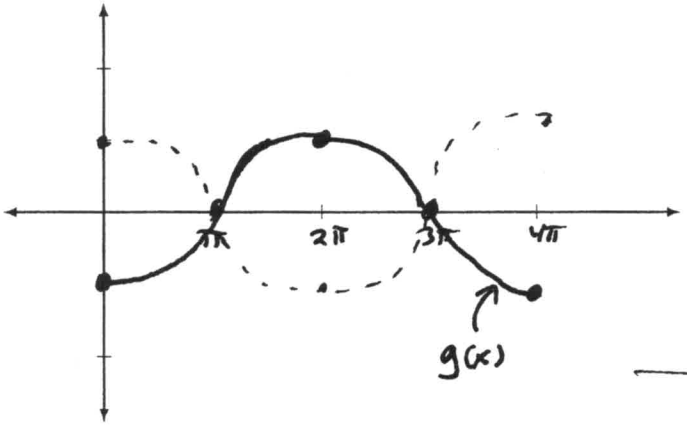


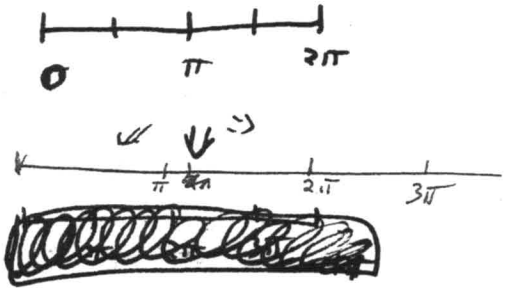
Period: 2π 1

(b) [6 points] $g(x) = -\cos(\frac{x}{2})$

Be sure you fill in the scale for the x and y axes.

Period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$





this is $\cos(\frac{x}{2})$ reflected across x-axis

(2) cos
(2) period 4π
(2) reflected

Period: 4π

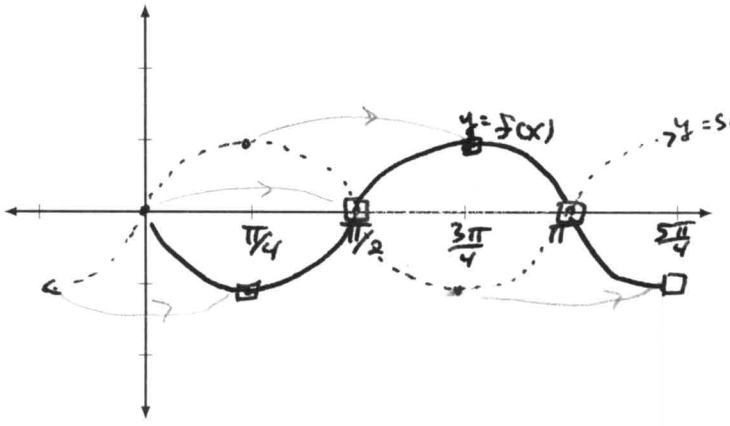
1 pt

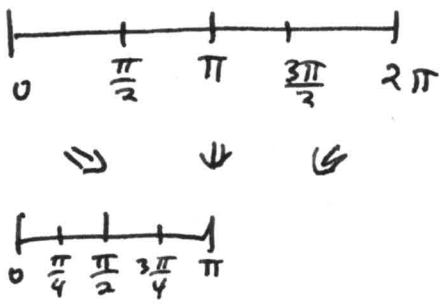
5. Graph the function below, and give its period.

(a) [6 points] $f(x) = \sin(2x - \pi) = \sin\left(2\left(x - \frac{\pi}{2}\right)\right)$ ← $\sin(2x)$ moved right $\frac{\pi}{2}$

Be sure you fill in the scale for the x and y axes.

Period = $\frac{2\pi}{2} = \pi$





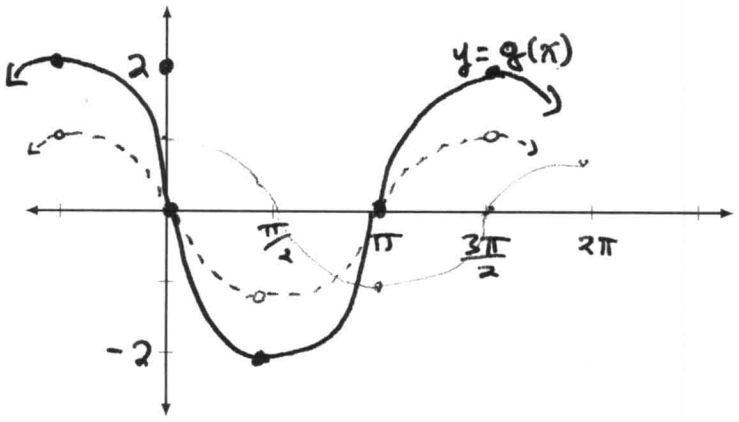
sine – 2 pt:
period π 2pt
right $\frac{\pi}{2}$ – 1pt

Period: π

(b) [6 points] $g(x) = 2\cos(x + \frac{\pi}{2})$

Be sure you fill in the scale for the x and y axes.

this is cos(x) 2pt
moved left $\frac{\pi}{2}$ 1pt
0 stretched by a factor of 2 2pt

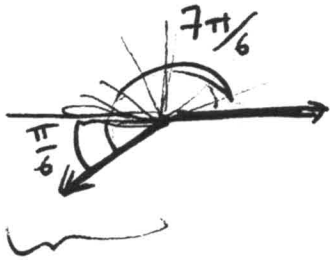


1pt

Period: 2π

6. [8 points] Compute all 6 trig functions of the angle

$$x = \frac{7\pi}{6}$$



Notice: $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\sin\left(\frac{7\pi}{6}\right) = \frac{-1}{2}$ 2 (1 sign, 1 #)

$\cos\left(\frac{7\pi}{6}\right) = \frac{-\sqrt{3}}{2}$ 2 (1 sign, 1 #)

$\tan\left(\frac{7\pi}{6}\right) = \frac{\frac{-1}{2}}{\frac{-\sqrt{3}}{2}}\left(\frac{-2}{-2}\right) = \frac{1}{\sqrt{3}}$ 1

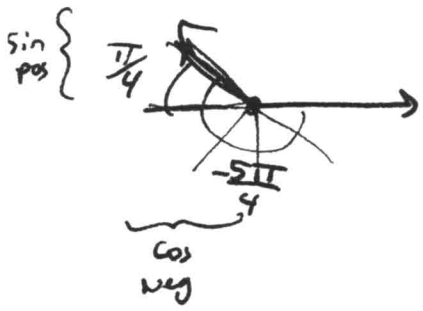
$\csc(x) = \frac{1}{\sin(x)} = -2$ 1

$\sec(x) = \frac{1}{\cos(x)} = \frac{-2}{\sqrt{3}}$ 1

$\cot(x) = \sqrt{3}$ 1

7. [8 points] Compute all 6 trig functions of the angle

$$x = \frac{5\pi}{4}$$



Recall $\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$$\sin\left(\frac{-5\pi}{4}\right) = \frac{+\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \qquad 2\left(\begin{smallmatrix}1\text{ sign}\\1\#\end{smallmatrix}\right)$$

$$\cos\left(\frac{-5\pi}{4}\right) = \frac{-\sqrt{2}}{2} = \frac{-1}{\sqrt{2}} \qquad 2\left(\begin{smallmatrix}1\text{ sign}\\1\#\end{smallmatrix}\right)$$

$$\tan\left(\frac{-5\pi}{4}\right) = \frac{\frac{\sqrt{2}}{2}}{\frac{-\sqrt{2}}{2}} = -\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1 \qquad 1$$

$$\csc(x) = \frac{1}{\sin(x)} = \frac{2}{\sqrt{2}} = \sqrt{2} \qquad 1$$

$$\sec(x) = \frac{1}{\cos(x)} = \frac{-2}{\sqrt{2}} = -\sqrt{2} \qquad 1$$

$$\cot(x) = \frac{1}{\tan(x)} = -1 \qquad 1$$

8. [10 points] You must **show your work** to receive credit.
Compute using the half angle formula:

$$\cos\left(\frac{5\pi}{12}\right)$$

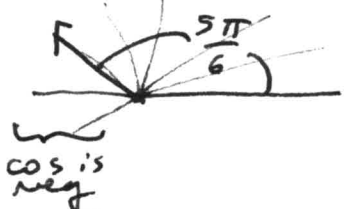
-2 if omit checking ±

if $\frac{u}{2} = \frac{5\pi}{12}$ 2pt

then $u = \frac{5\pi}{6}$

so $\cos\left(\frac{5\pi}{12}\right) = \pm\sqrt{\frac{1+\cos\left(\frac{5\pi}{6}\right)}{2}}$ 2pt

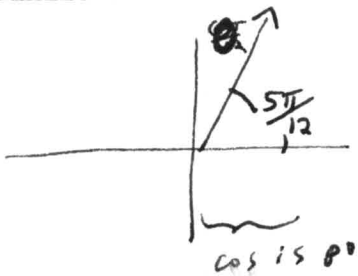
need: $\cos\left(\frac{5\pi}{6}\right)$



$\Rightarrow \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ 3pt

so $\cos\left(\frac{5\pi}{12}\right) = \pm\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}}$

5



$= +\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$ 3pt

8. _____

$= \sqrt{\frac{1}{2}\left(\frac{2-\sqrt{3}}{2}\right)} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

9. Remember: you must **show all work** to earn full credit.

(a) [6 points] Verify the following identity:

$$(\sin(x)+\cos(x))^2 = 1+\sin(2x)$$

$(\sin x+\cos x)(\sin x+\cos x)$

$(\sin(x)+\cos(x))^2 = \sin^2 x + 2\sin x\cdot\cos x + \cos^2(x)$ 2pt

$= \sin^2 x + \cos^2 x + 2\cdot\sin x\cdot\cos x$ 2pt

$= 1 + 2\cdot\sin x\cdot\cos x$ 2pt

$= 1 + \sin(2x)$ ✓

(b) [6 points] Verify the following identity:

$$\tan(x)+\cot(x) = \frac{\sec(x)}{\sin(x)}$$

$\tan(x)+\cot(x) = \frac{\sin(x)}{\sin(x)}\frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)}\cdot\frac{\cos(x)}{\cos(x)}$ 2pt

$= \frac{\sin^2(x)}{\sin(x)\cdot\cos(x)} + \frac{\cos^2(x)}{\sin(x)\cos(x)}$ 1pt

$= \frac{\sin^2(x)+\cos^2(x)}{\sin(x)\cdot\cos(x)}$ 2pt

$= \frac{1}{\sin(x)\cdot\cos(x)} = \frac{1}{\sin(x)}\cdot\frac{1}{\cos(x)} = \frac{1}{\sin(x)}\cdot\sec(x)$

1pt 1pt

$= \frac{\sec(x)}{\sin(x)}$

10. You must **show your work** to receive credit.

(a) [5 points] Solve the trigonometric equation for x in $[0, 2\pi]$:

$$\sin(2x) = \cos(x)$$

1pt $2 \cdot \sin(x) \cdot \cos(x) = \cos x$

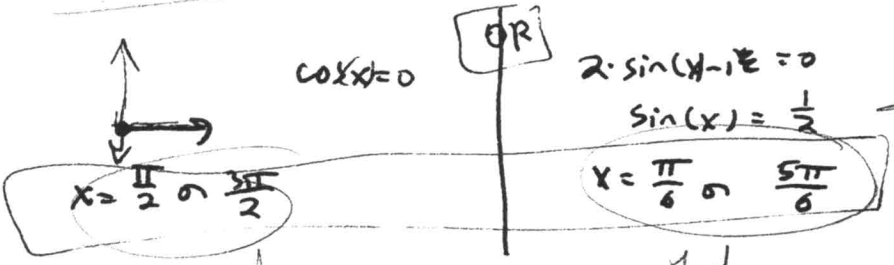
1 pt $2 \cdot \sin(x) \cdot \cos(x) - \cos(x) = 0$

1 pt $\cos(x)\left(2 \cdot \sin(x) - 1\right) = 0$

$\sin(\frac{\pi}{6}) = \frac{1}{2}$

$\cos x = 0$ OR $2 \cdot \sin(x) - 1 = 0$

$\sin(x) = \frac{1}{2}$



$x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ 1pt

$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ 1pt

(a) _____

(b) [5 points] Solve the trigonometric equation for x in $[0, 2\pi]$:

$$4\cos^2(x) - 1 = 0$$

$(2 \cdot \cos(x) + 1)(2 \cdot \cos(x) - 1) = 0$ 1pt

$2 \cdot \cos(x) + 1 = 0$

$\cos(x) = -\frac{1}{2}$

$\Leftrightarrow$

x is $\frac{4\pi}{3}$ 1

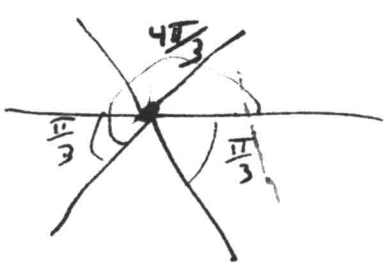
or $2\frac{\pi}{3}$ 1

$2 \cdot \cos(x) - 1 = 0$

$\cos(x) = \frac{1}{2}$

$\Leftrightarrow$

$x = \frac{\pi}{3}$ 1 or $\frac{5\pi}{3}$ 1



cos is neg cos is pos

(b) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ & $\frac{5\pi}{3}$

$\frac{3}{5}$ if correct solve for $\cos(x) = \frac{1}{2}$ but forget $\pm$