

Instructions:

- This exam contains 13 pages. When we begin, check you have *one* of each page.
- You will have 75 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided**.
In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:

By writing my name below, I agree that all the work
which appears on this exam is entirely my own.

I will not look at other peoples' work,
and I will not communicate with anyone else during the exam.

I will not use any calculators, notes, etc.

I understand that violating the above carries *serious consequences*,
both moral and academic.

Printed Name: Key Signature: _____

Section: _____

Page:	2	3	4	5	6	7	8	9	10	11	12	13	Total
Points:	10	10	10	10	10	8	10	6	6	10	4	6	100
Score:													

1. [5 points] True or false (1 point each – no work required):

(a) $(1 + \sqrt{x})^2 = 1 + x$

(a) False

(b) $(1 + x)^2 = 1 + x^2$

(b) False

(c) $(1 + x)^2 = 1 + 2x + x^2$

(c) True

(d) $\sqrt{1 + x} = 1 + \sqrt{x}$

(d) False

(e) $\frac{x-2}{x+3} = \frac{x}{x+3} - \frac{2}{x+3}$

(e) True

(f) $\frac{x-2}{x+3} = \frac{x-2}{x} + \frac{x-2}{3}$

(f) False

2. [5 points] Solve the following *equation* for x :

$$|3x + 1| = 2$$

$$\begin{array}{l} 3x+1 = 2 \\ 3x = 1 \\ x = \frac{1}{3} \end{array} \quad \Leftrightarrow \quad \begin{array}{l} \textcircled{+} \\ \textcircled{-} \end{array} \quad \begin{array}{l} 3x+1 = -2 \\ 3x = -3 \\ x = -1 \end{array}$$

2. $x = \frac{1}{3}$ or -1

3. [10 points] Find the set of x where the following *inequality* is true:

$$x^2 + 2x - 3 > 0$$

$$(x - 1)(x + 3) > 0$$

equality holds when
 $x = 1$ or $x = -3$



false when = holds
 \Rightarrow use open dots

Checking intervals

$$x = -4 \Rightarrow (-4 - 3)(-4 + 3) = (-7)(-1) \quad \checkmark$$

$$x = -1 \Rightarrow (-1 - 1)(-1 + 3) = (-2)(2) \quad \times$$

$$x = 2 \Rightarrow (2 - 1)(2 + 3) = (1)(5) \quad \checkmark$$

3. $(-\infty, -3) \cup (1, \infty)$

4. [10 points] Find the set of x such that the *inequality* is true:

$$\frac{x+3}{2x-4} \geq 0$$

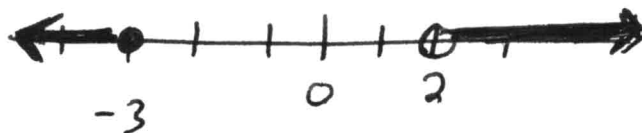
equality holds

when $\frac{x+3}{2x-4} = 0$

when $x+3=0$

when $x=-3$

(true when = holds
 \Rightarrow use solid dot)



undefined when

~~(2x-4)~~ $2x-4=0$

when $2x=4$

when $x=2$

(Plot with open dot)

check intervals:

$$x=-4 \Rightarrow \frac{(-4)+3}{2(-4)-4} = \frac{-1}{-12} \checkmark$$

$$x=0 \Rightarrow \frac{0+3}{2 \cdot 0 - 4} = \frac{3}{-4} \times$$

$$x=3 \Rightarrow \frac{3+3}{2 \cdot 3 - 4} = \frac{6}{2} \checkmark$$

4. $(-\infty, -3] \cup (2, \infty)$

5. [10 points] Find the set of x where the following *inequality* is true:

$$|3x + 1| \leq 2$$

equality holds when

$$|3x + 1| = 2$$

\Leftrightarrow

$$3x + 1 = 2 \quad \text{or} \quad 3x + 1 = -2$$

\Rightarrow

$$3x = -3$$

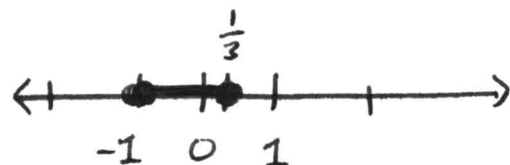
$$3x = 1$$

$$x = \frac{1}{3}$$

or

$$x = -1$$

true when = holds
 \Rightarrow use dot



check intervals:

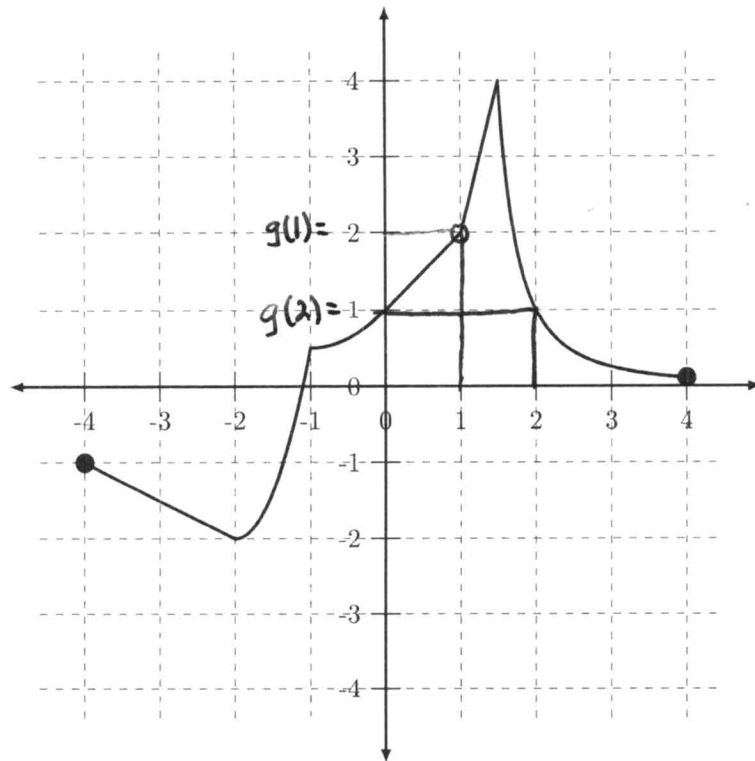
$$x = -2 \Rightarrow |3(-2) + 1| = |-6 + 1| = |-5| = 5 \quad X$$

$$x = 0 \Rightarrow |3 \cdot 0 + 1| = |1| = 1 \quad \checkmark$$

$$x = 1 \Rightarrow |3 \cdot 1 + 1| = |4| = 4 \quad X$$

5. $[-1, \frac{1}{3}]$

6. The graph of a function g is given below.



(a) [2 points] Explain why g is a function.

each input has one output
 it passes ^(or) the vertical line test

(b) [2 points] What is $g(1)$?

(b) 2

(c) [2 points] What is $g(2)$?

(c) 1

(d) [2 points] What is the domain of g ?

(d) $[-4, 4]$

(e) [2 points] What is the range of g ?

(e) $[-2, 4]$

7. [4 points] Find the equation for the line passing through the points (5,4) and (7,2). Write your answer in point-slope form and in slope-intercept form.

$$x_1 \quad y_1 \quad x_2 \quad y_2$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 4}{7 - 5} = \frac{-2}{2} = -1$$

$$y = m(x - x_1) + y_1$$

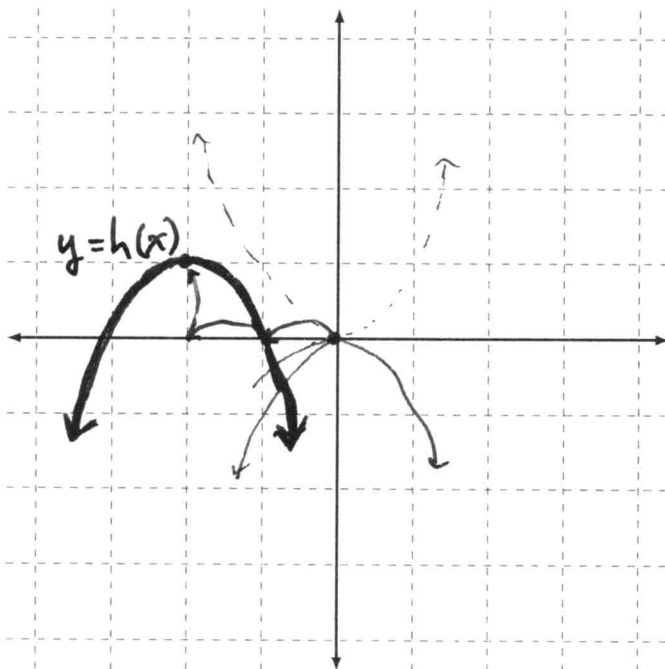
$$y = (-1)(x - 5) + 4 \quad \text{or} \quad y = (-1)(x - 7) + 2$$

$$y = -x + 5 + 4 \Rightarrow y = -x + 9$$

7. _____

8. [4 points] Sketch the graph of the following parabola. Write its vertex on the line below.

$$h(x) = -(x + 2)^2 + 1$$



$y = x^2$, flipped across x
 moved left 2
 up 1

8. vertex = (-2, 1)

9. [10 points] Completing the Square to get quadratic in the form $y = a(x - h)^2 + k$, and use this to sketch the graph of the function.

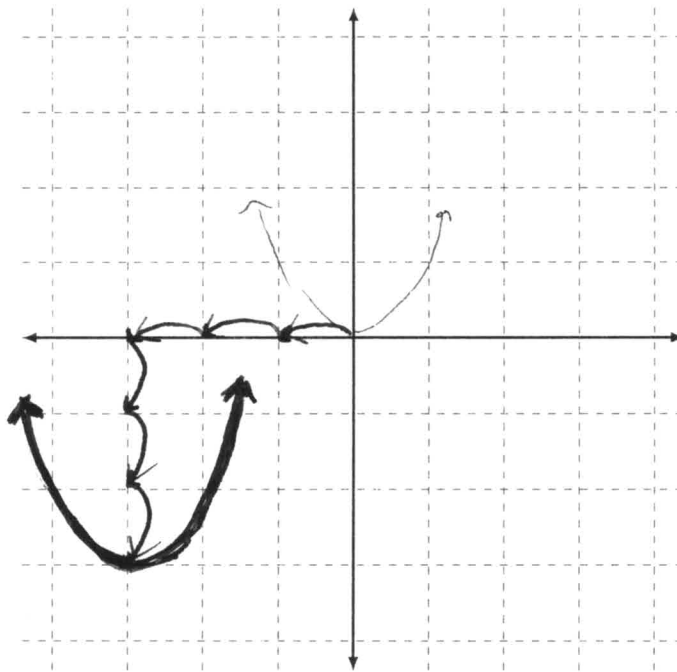
$$y = x^2 + 6x + 6$$

$$y = \left[x^2 + 6x + \frac{9}{1} \right] - \frac{9}{1} + 6$$

$$= (x + 3)(x + 3) - \frac{9}{1} + 6$$

$$= (x + 3)^2 - 3$$

9. $y = (x + 3)^2 - 3$



10. [6 points] Determine the domains of the following functions **Remember to show all work!**

(a) Find the domain of $f(x) = \sqrt{2x+4}$, and write it in interval notation.

$$f(x) \text{ is defined} \\ \Leftrightarrow$$

$$2x+4 \geq 0$$

$$\Leftrightarrow$$

$$2x \geq -4$$

$$\Leftrightarrow$$

$$x \geq -2$$



(a) $[-2, \infty)$

(b) Find the domain of $g(x) = x - 1$, and write it in interval notation.

$$g(x) \text{ is always defined}$$

(b) $(-\infty, \infty)$

(c) Graph the domain of $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{2x+4}}{x-1}$.

$$\left(\frac{f}{g}\right)(x) \text{ is defined} \Leftrightarrow$$

$$f(x) \text{ is defined}$$

AND

$$g(x) \text{ is defined}$$

$$g(x) \neq 0$$

$$\text{Both} \\ x \geq -2$$

AND

$$x-1 \neq 0$$

$$\text{Both} \\ x \geq -2$$

AND

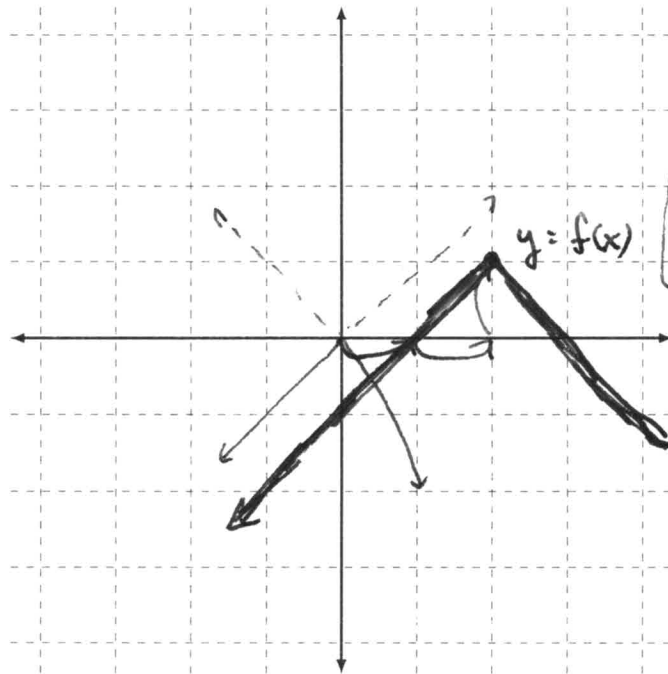
$$x \neq 1$$



(d) Write the domain of $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{2x+4}}{x-1}$ in interval notation.

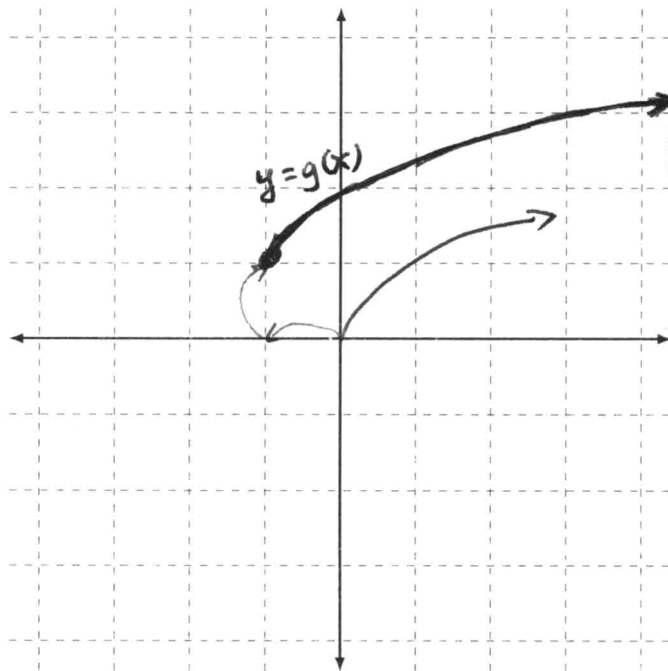
(d) $[-2, 1) \cup (1, \infty)$

11. [3 points] Sketch the graph of $f(x) = -|x - 2| + 1$



this is
 $y = |x|$
flipped across x -axis,
moved right 2
and up 1

12. [3 points] Sketch the graph of $g(x) = \sqrt{x + 1} + 1$



this is
 $y = \sqrt{x}$
moved left 1
and
up 1

13. [10 points] Graphing the polynomial

$$f(x) = (x+1)(x-1)(x-3) = x^3 - 3x^2 - x + 3$$

① eventual behavior:

$$x \text{ big} \Rightarrow f(x) \approx x^3$$



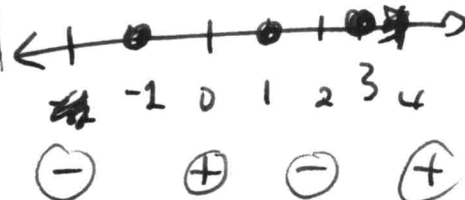
$$\textcircled{2} f(x) = 0$$

$$\Leftrightarrow$$

$$(x+1)(x-1)(x-3) = 0$$

$$\Leftrightarrow$$

$$x = -1 \text{ or } x = 1 \text{ or } x = 3$$

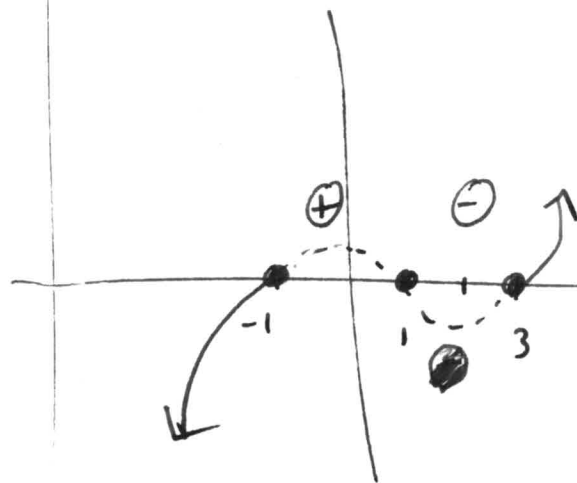
③ check intervals

$$x = -2 \Rightarrow (-2+1)(-2-1)(-2-3) \\ = (-1)(-3)(-5) \\ \ominus$$

$$x = 0 \Rightarrow (0+1)(0-1)(0-3) \\ = (1)(-1)(-3) \\ \oplus$$

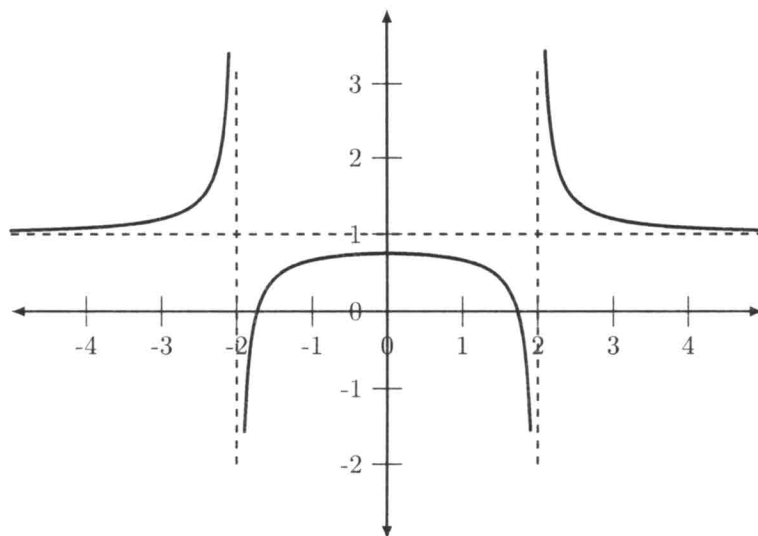
$$x = 2 \Rightarrow (2+1)(2-1)(2-3) \\ = (3)(1)(-1) \\ \ominus$$

$$x = 4 \Rightarrow (4+1)(4-1)(4-3) \\ = 5 \cdot 3 \cdot 1 \\ \oplus$$

Graphing:

14. [4 points] The graph of this function is given below

$$f(x) = \frac{x^2 - 3}{x^2 - 4}$$



(a) What are the horizontal asymptote(s) of $f(x)$?

(a) $y = 1$

(b) What are the vertical asymptote(s) of $f(x)$?

(b) $x = -2$ and $x = 2$

15. [6 points] Consider the following function:

$$f(x) = \frac{3x}{(x-2)^2} = \frac{3x}{x^2 - 4x + 4}$$

(a) Find the zero's of $f(x)$.

$$f(x) = 0$$

$$\Leftrightarrow$$

$$3x = 0$$

$$\Leftrightarrow$$

$$x = 0$$

(a) $x = 0$

(b) Find the horizontal asymptotes of $f(x)$.

when x is big

$$f(x) \approx \frac{3x}{x^2} = \frac{3}{x}$$

when x is big $f(x)$ is small

\Rightarrow asymptote at $y = 0$

(b) $y = 0$

(c) Find the vertical asymptotes of $f(x)$.

~~when~~ x is undefined

$$\Leftrightarrow$$

$$(x-2)^2 = 0$$

$$\Leftrightarrow$$

$$x - 2 = 0$$

$$\Leftrightarrow$$

$$x = 2$$

(c) $x = 2$