Key

Arithmetic with Fractions

Get a common denominator to add or subtract the following fractions.

(1) $\frac{x}{x}\cdot\frac{3}{5}+\frac{4}{x}\cdot\frac{5}{5} = \frac{3x}{5x}+\frac{20}{5x} = \frac{3x+20}{5x}$

(2) $\frac{1}{1}\cdot\frac{7}{x+2}-\frac{8}{1}\left(\frac{x+2}{x+2}\right) = \frac{7}{(x+2)}-\frac{8(x+2)}{(x+2)} = \frac{7-(8x+16)}{x+2} = \frac{7-8x-16}{x+2} = \frac{-9-8x}{x+2}$

(3) $1+\frac{1}{y}-\frac{1}{z} = \frac{yz}{yz}\cdot 1+\frac{z}{z}\cdot\frac{1}{y}-\frac{y}{y}\cdot\frac{1}{z} = \frac{yz}{yz}+\frac{z}{yz}-\frac{y}{yz} = \frac{yz+z-y}{yz}$

1. Multiplying Fractions

Multiply without simplifying.

(4) $\frac{3}{5}\cdot\frac{4}{x} = \frac{3\cdot 4}{5\cdot x} = \frac{12}{5\cdot x}$

(5) $\left(\frac{7}{x+2}\right)\left(\frac{8}{1}\right) = \frac{7\cdot 8}{x+2} = \frac{56}{x+2}$

(6) $\frac{3x}{7z}\cdot\frac{1}{x^2}\cdot\frac{4}{1} = \frac{3\cdot 4}{7}\cdot\frac{x}{x^2}\cdot\frac{1}{z} = \frac{12}{7}\cdot\frac{1}{x}\cdot\frac{1}{z} = \frac{12}{7\cdot x\cdot z}$

2. Dividing with Fractions

(7) $\frac{\frac{3}{5}}{\frac{4}{x}} = \frac{3}{5}\cdot\frac{x}{4} = \frac{3x}{20}$

(8) $\frac{7}{x+2}\div 8 = \frac{7}{x+2}\cdot\frac{1}{8} = \frac{7}{(x+2)\cdot 8} = \frac{7}{8x+16}$

(9) $\frac{\frac{1}{x}}{\frac{1}{x}} = \frac{1}{x} \cdot \frac{x}{1} = \frac{x}{x} = 1$

(10) $\frac{3x}{7z} \div (\frac{1}{x^2} \div 4) = \frac{3x}{7z} \div \left(\frac{1}{x^2} \cdot \frac{1}{4}\right) = \frac{3x}{7z} \div \left(\frac{1}{4x^2}\right) = \frac{3x}{7z} \cdot \frac{4x^2}{1} = \frac{12x^3}{7z}$

(11) Here's a good question for you: is $\frac{\frac{a}{b}}{c}$ the same as $\frac{a}{\frac{b}{c}}$?

$$\frac{\frac{a}{b}}{c} = \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{b \cdot c}$$

$$\frac{a}{\frac{b}{c}} = a \cdot \frac{c}{b} = \frac{a \cdot c}{b}$$

these are not the same!

3. Canceling in Fractions

Cancel as much as possible. If it's not possible to cancel anything, say so.

(12) $\frac{3x^2 + x}{x} = \frac{(3x+1)x}{x} = \frac{3x+1}{1}$

(13) $\frac{3 + 6x}{x} = \frac{3(1+2x)}{x}$ cannot be simplified

(14) $\frac{3 + 6x}{3} = \frac{3(1+2x)}{3} = 1+2x$

(15) $\frac{3 + 6x}{1 + x} = \frac{3(1+2x)}{1+x}$ cannot be simplified

(16) $\frac{3 + 6x}{1 + 2x} = \frac{3\cancel{(1+2x)}}{\cancel{1+2x}} = 3$

(17) $\frac{3 + 6x}{3 - 6x} = \frac{3(1+2x)}{3(1-2x)}$ cannot be simplified.

(18) $\frac{3+6x}{-3-6x}$ $= \frac{3\cancel{(1+2x)}}{-3\cancel{(1+2x)}} = \frac{\cancel{3}}{-\cancel{3}} = -1$

(19) $\frac{x+y}{y}$ [redacted]

$= (x+y)\frac{1}{y} = \frac{x}{y} + \frac{y}{y} = \frac{x}{y} + 1$

(20) $\frac{b}{b-a}$ CANNOT be simplified

(21) $\frac{3a^2-2a}{xa}$ $= \frac{\cancel{a}(3a-2)}{x \cdot \cancel{a}} = \frac{3a-2}{x} = \frac{3a}{x} - \frac{2}{x}$

(22) $\frac{4y-8}{y-2}$ $= \frac{4\cancel{(y-2)}}{\cancel{(y-2)}} = 4$

(23) $\frac{x-2}{x^2-4}$ $= \frac{\cancel{x-2}}{(x+2)\cancel{(x-2)}} = \frac{1}{x+2}$

4. Summary

You've practiced all of these techniques one at a time. Can you combine them?

(24) Simplify $\frac{\frac{1}{x+h} - \frac{1}{x}}{h}$. $= \frac{1}{h}\left(\frac{1}{(x+h)} \cdot \frac{x}{x} - \frac{1}{x} \cdot \left(\frac{x+h}{x+h}\right)\right) = \frac{1}{h}\left(\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}\right)$

$= \frac{1}{h}\left(\frac{x-(x+h)}{x(x+h)}\right)$

$= \frac{1}{h}\left(\frac{x-x-h}{x(x+h)}\right)$

$= \frac{1}{h}\left(\frac{-h}{x(x+h)}\right) = \frac{-1}{x(x+h)}$

(25) Simplify $\frac{1}{x+2} - \frac{1}{x-1}$.

$= \frac{1}{(x+2)} \cdot \left(\frac{x-1}{x-1}\right) - \frac{1}{(x-1)} \cdot \left(\frac{x+2}{x+2}\right)$

$= \frac{x-1}{(x+2)(x-1)} - \frac{x+2}{(x-1)(x+2)}$

$= \frac{(x-1)-(x+2)}{(x+2)(x-1)} = \frac{x-1-x-2}{(x+2)(x-1)} = \frac{-3}{(x+2)(x-1)}$

(26) Simplify $\frac{x+\frac{1}{x-2}}{x}$. $= \dfrac{\frac{x(x-2)}{(x-2)} + \frac{1}{(x-2)}}{x} = \dfrac{\frac{x(x-2)+1}{(x-2)}}{x} = \frac{1}{x}\left(\frac{x^2-2x+1}{x-2}\right)$

$= \frac{1}{x}\,\frac{(x-1)^2}{(x-2)}$

(27) Simplify $\frac{2}{x}+\frac{1}{(x-1)^2}$ $= \frac{2(x-1)^2}{x(x-1)^2} + \frac{1\cdot x}{x(x-1)^2} = \frac{2(x-1)^2+x}{x(x-1)^2}$

$= \frac{2(x^2-2x+1)+x}{x(x-1)^2} = \frac{2x^2-4x+2+x}{x(x-1)^2} = \frac{2x^2-3x+2}{x(x-1)^2}$ [redacted]

(28) Simplify $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$. $= \frac{x\cdot xz}{y\cdot xz} + \frac{y}{z}\cdot\frac{xy}{xy} + \frac{z}{x}\cdot\frac{yz}{yz}$

$= \frac{x^2z}{xyz} + \frac{xy^2}{xyz} + \frac{yz^2}{xyz} = \frac{x^2z+xy^2+yz^2}{xyz}$

(29) Simplify $\frac{a-b}{\frac{a}{b}-\frac{b}{a}}$ $= \dfrac{a-b}{\left(\frac{a}{b}\cdot\frac{a}{a} - \frac{b}{a}\cdot\frac{b}{b}\right)} = \dfrac{a-b}{\left(\frac{a^2}{ab} - \frac{b^2}{ab}\right)} = \dfrac{a-b}{\left(\frac{a^2-b^2}{ab}\right)}\cdot\frac{ab}{ab}$

$= \frac{ab(a-b)}{a^2-b^2} = \frac{ab\cancel{(a-b)}}{(a+b)\cancel{(a-b)}} = \frac{ab}{a+b}$

(30) True or false? $\frac{x}{x+y} = 1+\frac{x}{y}$.

FALSE! eg: $\frac{2}{2+3} = \frac{2}{5}$ but $1+\frac{2}{3} = \frac{5}{3}$

(31) True or false? $\frac{x+y}{x} = 1+\frac{y}{x}$.

$\frac{x+y}{x} = \frac{1}{x}(x+y) = \frac{x}{x}+\frac{y}{x} = 1+\frac{y}{x}$

TRUE!

Rules of Exponents

Simplify the following:

(1) $x^3 \cdot x^7 \cdot (x^3)^5$ $= x^{3+7} \cdot x^{3\cdot 5} = x^{10} \cdot x^{15} = x^{10+15} = x^{25}$

(2) $(x^{-3})(x^h)^2(x)$ $= \frac{1}{x^3} \cdot x^{2h} \cdot x = \frac{x^{2h+1}}{x^3} = x^{(2h+1)-3} = x^{2h-2}$

(3) $q^{-4} \cdot \frac{4q}{(q^{-2})^3}$ $= \frac{1}{q^4} \cdot \frac{4 \cdot q^1}{q^{-6}} = \frac{1}{q^4} \cdot 4q^{1-(-6)}$ [redacted] $= \frac{1}{q^4} \cdot 4q^7$ [redacted]

$\boxed{= 4q^3}$

(4) $\frac{3}{y^2}\frac{y}{y^{-8}}$ $=$ [redacted] $\frac{3}{y^2} \cdot y^{1-(-8)} = \frac{3}{y^2}\, y^{1+8} = 3\,\frac{y^9}{y^2} = 3y^7$

(5) $\left(\frac{2y}{x}\right)^2$ $= \frac{2^2 \cdot y^2}{x^2} = \frac{4y^2}{x^2}$

(6) $\frac{a}{a^4}\left(\frac{3}{a^2}\right)^{-3}$ $= \frac{a}{a^4} \cdot \left(\frac{a^2}{3}\right)^3 = \frac{a \cdot a^{2\cdot 3}}{a^4 \cdot 3^3} =$ [redacted] $= \frac{a^7}{a^4 \cdot 3^3} = \boxed{\frac{a^3}{3^3}}$

(7) $z^{-\frac{1}{3}}z^{0.5}$ $= z^{-\frac{1}{3}}\, z^{\frac{1}{2}} = z^{-\frac{1}{3}+\frac{1}{2}} = z^{\frac{-2}{6}+\frac{3}{6}} = z^{\frac{1}{6}}$ [redacted]

(8) $(x^{\frac{1}{2}})^{\frac{1}{3}}x^{\frac{1}{4}}$ $= x^{\frac{1}{2}\cdot\frac{1}{3}}\, x^{\frac{1}{4}} = x^{\frac{2}{2}\cdot\frac{1}{2\cdot 3} + \frac{1}{4}\cdot\frac{3}{3}} = x^{\frac{2}{12}+\frac{3}{12}} = x^{5/12}$

(9) $(x^{\sqrt{2}})^{\sqrt{2}} = x^{\sqrt{2}\cdot\sqrt{2}} = x^{2}$

(10) $\left(\frac{1}{x}\right)^{-\frac{1}{2}} = \left(\frac{x}{1}\right)^{\frac{1}{2}} = \sqrt{x}$

(11) $\left(\frac{1}{x^{-2}}\right)^{\frac{1}{2}} = \left(\frac{x^{2}}{1}\right)^{\frac{1}{2}} = \sqrt{x^{2}} = x$

(12) $\left(\frac{x^{-\frac{1}{2}}}{x^{2}}\right)^{2} = \frac{x^{-\frac{1}{2}\cdot 2}}{x^{2\cdot 2}} = \frac{x^{-1}}{x^{4}} = \frac{1}{x}\cdot\frac{1}{x^{4}} = \frac{1}{x^{5}}$

(13) $\frac{(x+2)(x-3)}{x(x+2)^{-2}(x-3)^{1/2}} = \frac{(x+2)(x+2)^{2}\ (x-3)}{x\ \text{[redacted]}\ (x-3)^{1/2}} = \frac{(x+2)^{3}}{x}\cdot(x-3)^{1-1/2}$

$= \frac{(x+2)^{3}}{x}(x-3)^{\frac{1}{2}}$

(14) $(x+1)^{1-\pi}(x+1)^{1+\pi} =$ [redacted] $(x+1)^{(1-\pi)+(1+\pi)} = (x+1)^{2}$

(15) $\left(\frac{x^{4/3}x^{3}}{x^{-2}}\right)^{3}\left(\frac{x^{-1}}{x}\right) = \left(\frac{x^{4}\cdot x^{9}}{x^{-2\cdot 3}}\right)\cdot\left(x^{-1-1}\right) = \frac{x^{13}}{x^{-6}}\,x^{-2} = x^{13-(-6)+(-2)}$

$= x^{13+6-2} = x^{17}$

(16) $\left(\frac{(x-1)^{-2}(x+1)^{3}}{(x^{2}-1)}\right)^{2} = \frac{(x-1)^{-4}\ (x+1)^{6}}{\left((x+1)(x-1)\right)^{2}} = \frac{(x+1)^{6}}{(x-1)^{4}\,(x+1)^{2}(x-1)^{2}} = \frac{(x+1)^{4}}{(x-1)^{6}}$

$(x+1)(x-1)$

Solving Equations

(1) $\frac{y}{3} + 4 = \frac{y}{2}$

$4 = \frac{3}{3}\cdot\frac{y}{2} - \frac{y}{3}\cdot\frac{2}{2}$

$4 = \frac{3y}{6} - \frac{2y}{6}$

$4 = \frac{3y - 2y}{6}$

$4 = \frac{y}{6}$

$\boxed{24 = y}$

(2) $\left(\frac{2}{3}x + 3(x-1)\right) = (8)3$

$2x + 9(x-1) = 24$

$2x + 9x - 9 = 24$

$11x = 33$

$\boxed{x = 3}$

(3) $3 + 2x = 2(\frac{3}{2} + x)$

$3 + 2x = 3 + 2x$ ← this is true for all x!

(4) $\frac{1}{x} = \frac{3}{x} + 1$

method 1

$\frac{1}{x} - \frac{3}{x} = 1$

$\frac{1-3}{x} = 1$

$\frac{-2}{x} = 1$

$\boxed{-2 = x}$

method 2: mult both sides by x

$1 = 3 + x$

$\boxed{-2 = x}$

(5) $\frac{3}{5}x + 8 = -x + \frac{1}{5}(2 + 8x)$

$5\left(\frac{3}{5}x + 8\right) = \left(-x + \frac{2}{5} + \frac{8x}{5}\right)5$

$3x + 40 = -5x + 2 + 8x$

$40 = 2$ ← NOT true for any x

⇒ the equation has no solutions

(6) $\sqrt{x} = 2 - x$

to get rid of $\sqrt{x}$, square both sides

$x = (2-x)^2$

$x = (2-x)(2-x)$

$x = 4 - 2x - 2x + x^2$

$x = 4 - 4x + x^2$

$0 = 4 - 5x + x^2$

$x^2 - 5x + 4 = 0$

$0 = (x-4)(x-1)$

multiply to get 4

add to get -5

this is true when $x = 4$ or $x = 1$

But the origional equation is only true for $x = 1$

(7) $\frac{2z-1}{z+2} = \frac{4}{5}$

[redacted]

$5(z+2)\left(\frac{2z-1}{z+2}\right) = \frac{4}{5}\cdot(z+2)\cdot 5$

$5(2z-1) = 4(z+2)$

$10z - 5 = 4z + 8$

$6z = 13$

$z = \frac{13}{6}$

(8) $y = 1 + \sqrt{2-2y}$

[redacted] $y-1$ [redacted] $= \sqrt{2-2y}$

$(y-1)(y-1) = (y-1)^2 = 2-2y$

$y^2 - 2y + 1 = 2 - 2y$

$y^2 - 1 = 0$

$(y+1)(y-1) = 0$

this is true when $y = 1$ or $y = -1$

But the [redacted] origional equation is only true when $y = 1$

(9) $(x+2)^2 = 4$

$(x^2+2)(x+2) =$ [redacted] 4

$x^2 + 4x + 4 = 4$

$x^2 + 4x = 0$

$x(x+4) = 0$

$\boxed{x = 0 \text{ or } x = -4}$

(10) $\frac{1}{x-1} + \frac{1}{x+2} = -\frac{8}{5}$

$\left(\frac{x+2}{x+2}\right)\frac{1}{(x-1)} + \frac{1}{x+2}\left(\frac{x-1}{x-1}\right) = \frac{-8}{5}$

$\frac{x+2}{(x+2)(x-1)} + \frac{x-1}{(x+2)(x-1)} = \frac{-8}{5}$

$\frac{2x+1}{(x+2)(x-1)} = \frac{-8}{5}$

cross multiplying gives:

$-8(x+2)(x-1) = 5(2x+1)$

$-8(x^2+x-2) = 10x + 5$

$-8x^2 - 8x + 16 = 10x + 5$

$0 = 8x^2 + 18x -$ [redacted] 11

[redacted]

$0 = (4x + 11)(2x - 1)$

$x = -$[redacted]$\frac{11}{4}$ or $x = \frac{1}{2}$

$22 - 4 = 18$

(11) Solve for x:

$$\frac{a+x}{b-x} = c$$

$a + x = c\cdot(b-x)$

$a + x = cb - cx$

$x + cx = cb - a$

$x(1+c) = cb - a$

$x = \frac{cb-a}{1+c}$

(12) Solve for a:

$$\frac{a+x}{b-x} = c$$

$a + x = c(b-x)$

$a + x = cb - cx$

$a = cb - cx - x$

(13) Solve for θ:

$a x^2 + bx + c = 0$

$\Leftrightarrow$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x\theta^2 + \theta - 2 = 0$$

$$\theta = \frac{-1 \pm \sqrt{1 - 4 \cdot x (-2)}}{2 \cdot x}$$

(14) Solve for $\frac{dy}{dx}$:

$$3y^2 \boxed{\frac{dy}{dx}} + 2y + 2x \boxed{\frac{dy}{dx}} = 7$$

$$(3y^2 + 2x) \cdot \frac{dy}{dx} + 2y = 7$$

$$(3y^2 + 2x) \cdot \frac{dy}{dx} = 7 - 2y$$

$$\frac{dy}{dx} = \frac{7 - 2y}{3y^2 + 2x}$$

Solving Inequalities

Solve the following inequalities:

(1) $x + 5 < 1$

$x < 1 - 5$

$x <$ [redacted] -4

(2) $1 - 2x \geq 0$

$1 \geq 2x$

$\frac{1}{2} \geq x$

OR

$-2x \geq -1$

[redacted] $x \leq \frac{1}{2}$

(3) $3 + \frac{x}{2} < 6$

$\frac{x}{2} < 6 - 3$

$\frac{x}{2} < 3$

$x < 6$

(4) $4 - 10x < 2(1 - x)$

$4 - 10x < 2 - 2x$

$2 < 8x$

$\frac{1}{4} < x$

(5) $-5x + 1 \leq 2 - x$

$1 \leq 2 - x + 5x$

$1 - 2 \leq 4x$

$-1 \leq 4x$

$-\frac{1}{4} \leq x$

(6) $2x + 1 > \frac{7}{2} + \frac{4}{3}x$

$2 \cdot 3(2x+1) > 2 \cdot 3\left(\frac{7}{2} + \frac{4}{3}x\right)$

$12x + 6 > 21 + 8x$

$12x - 8x > 21 - 6$

$4x >$ [redacted] 15 [redacted]

$x > \frac{15}{4}$

(7) $3 < \frac{x+2}{3} \leq 5$

$9 < x + 2 \leq 15$

$7 < x \leq 13$

(8) $4 > 2 - x > 3$

$-4 < -2 + x < -3$

$-2 < x < -1$

this is when false holds =

(9) $\frac{x^2-9}{x+1} < 0$

Step 1:
equality holds when

$\frac{(x+3)(x-3)}{(x+1)} = 0$ which is when $x = 3$ or $x = -3$

$\frac{(x+3)(x-3)}{(x+1)} < 0$

Step 2:
LHS is undefined when $x = -1$

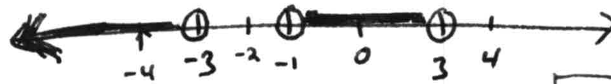
step 3:

$x = -4 \Rightarrow \frac{(-1)(-7)}{-3} = \,<0$ our inequality TRUE [redacted]

$x = -2 \Rightarrow \frac{(1)(-5)}{(-1)} > 0$ ... is FALSE

$x = 0 \Rightarrow \frac{(3)(-3)}{1} < 0$ ... is TRUE

$x = 4 \Rightarrow \frac{(7)(1)}{5} > 0$ False



[redacted] true for x in $(-\infty,-3) \cup (-1,3)$

(10) $x^2 + 5x + 1 < -5$ this is false when = holds

$x^2 + 5x + 6 < 0$

$(x+2)(x+3) < 0$

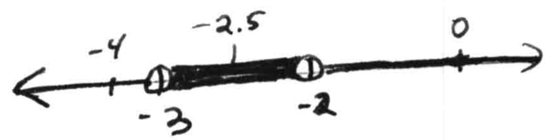
Step 1 equality holds when $x=-2$ [redacted] $x=-3$

Step 2 LHS always defined

step 3: $x = -4 \Rightarrow (-2)(-1) > 0 \Rightarrow$ don't shade

$x = -2.5 \Rightarrow (-.5)(.5) < 0 \Rightarrow$ shade

$x = 0 \Rightarrow (2)(3) > 0 \Rightarrow$ don't shade



true for x in $(-3,-2)$

(11) $\frac{(x-2)(x+4)}{x+7} > 0$ this is false when = holds

equality holds when $x = 2, -4$.

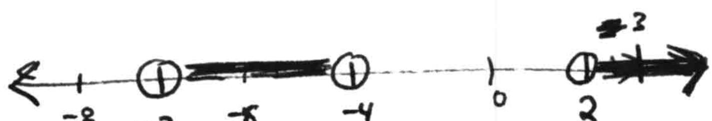
LHS undefined when $x = -7$

$x = 3 \Rightarrow \frac{(1)(7)}{10} > 0$ TRUE!

$x = 0 \Rightarrow \frac{(-2)(4)}{7} < 0$ FALSE

$x = -6 \Rightarrow \frac{(-8)(-2)}{(1)} > 0$ TRUE!

$x = -8 \Rightarrow \frac{(-10)(-4)}{(-1)} < 0$ FALSE



true for x in $(-7,-4) \cup (2,\infty)$

(12) $\frac{(x-2)(x+4)}{x+7} \geq 0$ TRUE when = holds!

equality holds when $x = 2, -4$

LHS undefined when $x = -7$

#'s in Intervals are the same as in (11)

the inequality is Now TRUE when = holds

true for x in $(-7,-4] \cup [2,\infty)$