

Name: Solutions

Section: \_\_\_\_\_

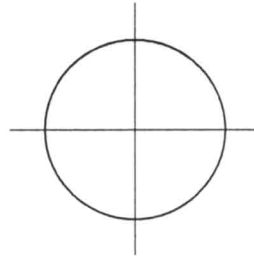
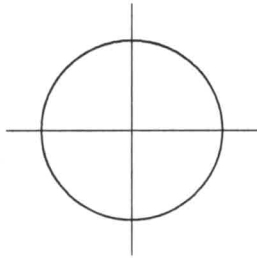
You can find examples of the earlier material in the book homework, in the class notes, and in the quizzes. Quizzes #5 and 6 cover Basic Trigonometry. Quiz #7 covers Trigonometric Functions.

Now that we have had time to practice, the exam may contain some problems that are more difficult than the ones on the quizzes.

## Basic Trigonometry

1. Be able to plot all sorts of angles on the unit circle.

E.g.  $-\frac{4\pi}{3}$ ,  $\frac{11\pi}{4}$ ,  $\frac{21\pi}{2}$ ,  $-\frac{9\pi}{4}$ , etc.



2. Convert  $60^\circ$  and  $30^\circ$  to radians.

Answer: \_\_\_\_\_

3. Fix the circle with radius  $1''$ .

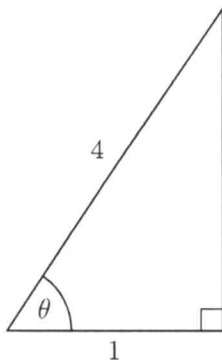
Find the length of the arc intercepted by the angle  $\theta = 3$  radians.

Answer: \_\_\_\_\_

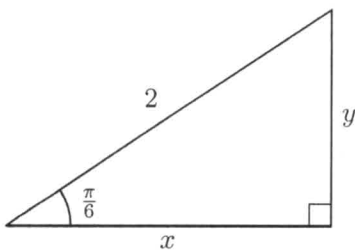
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1. Find the value of all six trigonometric functions of the angle  $\theta$ .



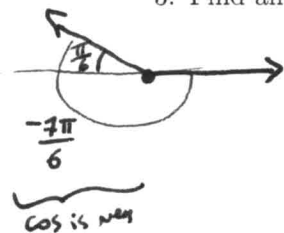
2. Find the lengths of the missing sides of the triangle.



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3. Find all six trigonometric functions of the angle  $t = -\frac{7\pi}{6}$ .



$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \begin{cases} \sin\left(-\frac{7\pi}{6}\right) = \frac{1}{2} \\ \cos\left(-\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2} \\ \tan\left(-\frac{7\pi}{6}\right) = \frac{\sin\left(-\frac{7\pi}{6}\right)}{\cos\left(-\frac{7\pi}{6}\right)} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} \end{cases}$$

$$\csc(t) = \frac{1}{\sin(t)} = 2$$

$$\sec(t) = \frac{1}{\cos(t)} = -\frac{2}{\sqrt{3}}$$

$$\cot(t) = \frac{1}{\tan(t)} = -\sqrt{3}$$

4. Find all six trigonometric functions of the angle  $\theta = \frac{11\pi}{4}$ .

5. Find all six trigonometric functions of the angle  $x = \frac{5\pi}{3}$ .

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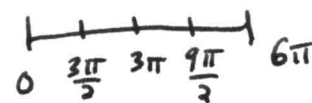
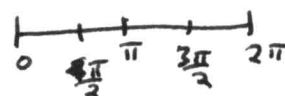
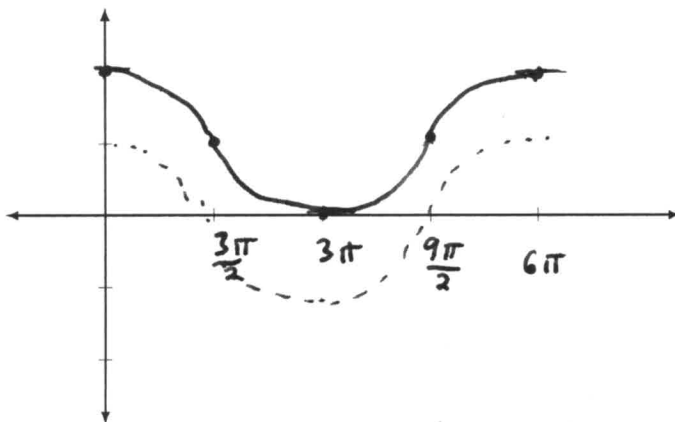
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### Trigonometric Functions

1. Sketch the graph of  $f(x) = \cos(\frac{x}{3}) + 1$ , and write its period on the line below.

Be sure you fill in the scale for the  $x$  and  $y$  axes.

Period =  $\frac{2\pi}{\frac{1}{3}} = 6\pi$



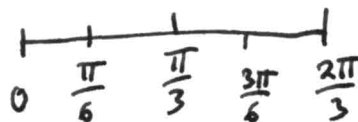
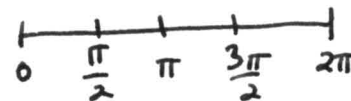
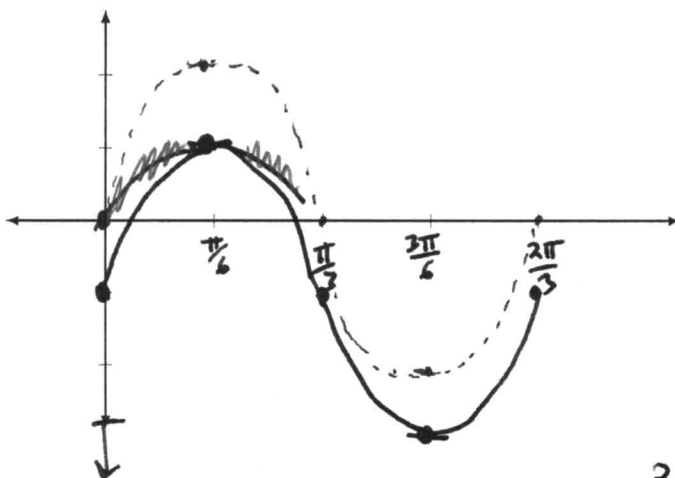
Period: 6π

Amplitude: 1

2. Sketch the graph of  $g(x) = 2\sin(3x) - 1$ , and write its period on the line below.

Be sure you fill in the scale for the  $x$  and  $y$  axes.

Period =  $\frac{2\pi}{3}$



Period:  $\frac{2\pi}{3}$

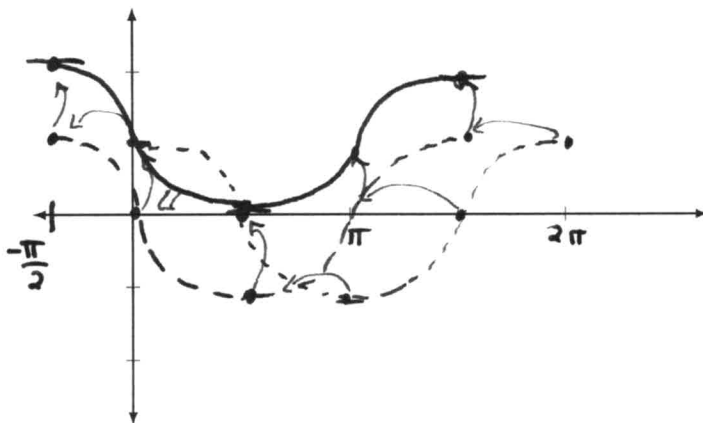
Amplitude: 2

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3. Sketch the graph of  $f(x) = \cos(x + \frac{\pi}{2}) + 1$ , and write its period on the line below.

Be sure you fill in the scale for the  $x$  and  $y$  axes.



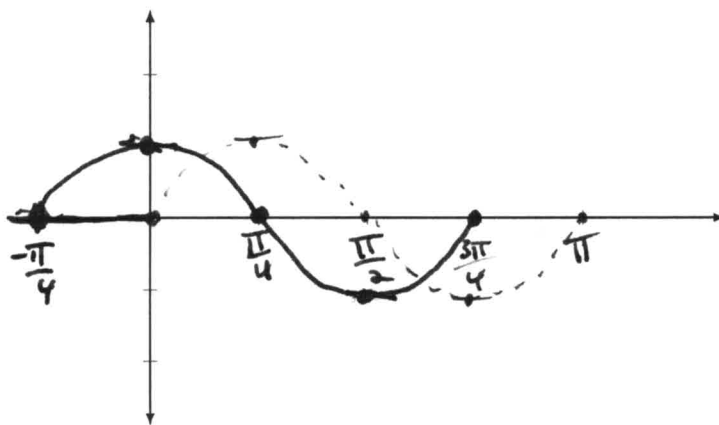
$\cos(x)$   
 moved left  $\frac{\pi}{2}$   
 and up 1

Period: 2π

Amplitude: 1

4. Sketch the graph of  $g(x) = \sin(2x + \frac{\pi}{4})$ , and write its period on the line below.

Be sure you fill in the scale for the  $x$  and  $y$  axes.



$\sin(2(x + \frac{\pi}{4}))$   
 is  $\sin(2x)$   
 moved left  $\frac{\pi}{4}$   
 period =  $\frac{2\pi}{2} = \pi$

Period: π

Amplitude: 1

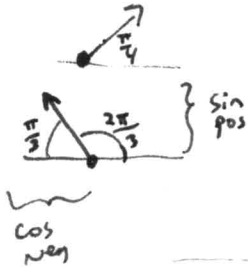
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## Trigonometric Equations

1. Use the sum of angles formulas to compute

$$(a) \sin\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{2\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right) \cdot \sin\left(\frac{2\pi}{3}\right)$$



$$= \frac{\sqrt{2}}{2} \cdot \frac{-1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

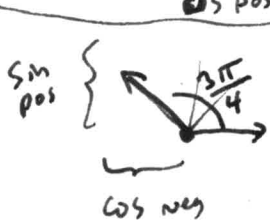
$$(b) \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \cos\left(\frac{\pi}{4}\right) \cos\left(\frac{2\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right) \sin\left(\frac{2\pi}{3}\right)$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{-1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{-\sqrt{2} - \sqrt{6}}{4}$$

2. Use the half angle formula to compute

$$(a) \sin\left(\frac{3\pi}{8}\right) = \pm \sqrt{\frac{1 - \cos\left(\frac{3\pi}{4}\right)}{2}} = \pm \sqrt{\frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2}} = \pm \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}}$$



$$\frac{x}{2} = \frac{3\pi}{8}$$

$$x = \frac{3\pi}{4}$$

$$\sin\left(\frac{3\pi}{8}\right) \text{ is positive} \Rightarrow \sin\left(\frac{3\pi}{8}\right) = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}}$$



$$(b) \cos\left(\frac{3\pi}{8}\right) = \pm \sqrt{\frac{1 + \cos\left(\frac{3\pi}{4}\right)}{2}}$$

$$= \pm \sqrt{\frac{1 + \left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= \pm \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$$

 $\cos\left(\frac{3\pi}{8}\right)$  is positive

$$\Rightarrow \cos\left(\frac{3\pi}{8}\right) = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$$

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3. Verify the trigonometric identity:

$$\cos(x) + \sin(x) \tan(x) = \sec(x)$$

$$\begin{aligned} \cos(x) + \sin(x) \tan(x) &= \frac{\cos(x)}{\cos(x)} \cdot \cos(x) + \sin(x) \cdot \frac{\sin(x)}{\cos(x)} \\ &= \frac{\cos^2(x)}{\cos(x)} + \frac{\sin^2(x)}{\cos(x)} \\ &= \frac{\cos^2(x) + \sin^2(x)}{\cos(x)} \\ &= \frac{1}{\cos(x)} \\ &= \sec(x) \quad \checkmark \end{aligned}$$

4. Verify the following trigonometric identity:

$$\csc(2x) = \frac{\csc(x)}{2 \cos(x)}$$

$$\begin{aligned} \csc(2x) &= \frac{1}{\sin(2x)} = \frac{1}{2 \cdot \sin(x) \cdot \cos(x)} \\ &= \frac{1}{2 \cdot \cos(x)} \cdot \frac{1}{\sin(x)} \\ &= \frac{\frac{1}{\sin(x)}}{2 \cdot \cos(x)} \\ &= \frac{\csc(x)}{2 \cdot \cos(x)} \end{aligned}$$

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5. Solve the trigonometric equation: for  $x$  in  $[0, 2\pi]$ 

$$2\sin^2(x) - \sin(x) - 1 = 0$$

$$(2\sin(x) + 1)(\sin(x) - 1) = 0$$

either

$$2\sin(x) + 1 = 0$$

$$\sin(x) = -\frac{1}{2}$$

$$\Leftrightarrow$$

$$x = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}$$

$$\sin(x) - 1 = 0$$

$$\sin(x) = 1$$

$$\Leftrightarrow$$

$$x = \frac{\pi}{2}$$

$$\Leftrightarrow$$

$$x \text{ is } \frac{\pi}{2}, \frac{7\pi}{6}, \text{ or } \frac{11\pi}{6}$$

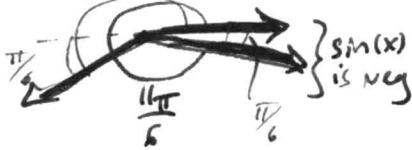
think:

$$2a^2 - a - 1 = 0$$

$$(2a + 1)(a - 1) = 0$$

Remember:

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

 $\frac{7\pi}{6}$ 6. Solve the trigonometric equation: for  $x$  in  $[0, 2\pi]$ 

$$2\sin^2(x) = 2 + \cos(x)$$

$$2(1 - \cos^2(x)) = 2 + \cos(x)$$

$$2 - 2\cos^2(x) = 2 + \cos(x)$$

$$-2$$

$$-2$$

$$-2\cos^2(x) = \cos(x)$$

$$\Leftrightarrow$$

$$0 = \cos(x) + 2\cos^2(x)$$

$$\Leftrightarrow$$

$$0 = \cos(x)(1 + 2\cos(x))$$

either

$$\cos(x) = 0$$

$$x = 0 \text{ or } \pi \text{ or } 2\pi$$

$$\text{or}$$

$$1 + 2\cos(x) = 0$$

$$\cos(x) = -\frac{1}{2}$$

$$\Leftrightarrow$$

$$x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$$

$$\Leftrightarrow$$

$$x = 0, \pi, 2\pi, \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$$

