Exam I Review Worksheet Math 1060 Class #11

Name: ______________ Section: ______________

Solving Equations

1. Find the values of x where the equality is true:

$$x^3 + 4x^2 + 3x = 0$$

$$x(x^2+4x+3)=0$$

$$x(x+3)(x+1)=0$$

$$\Leftrightarrow$$

$$x=0 \quad \underline{\text{or}} \quad x=-3 \quad \underline{\text{or}} \quad x=-1$$

2. Find the values of x where the equality is true:

$$x^2 + 4x + 4 = 1$$

$$\Leftrightarrow$$

$$x^2+4x+3=0$$

$$\Leftrightarrow$$

$$(x+3)(x+1)=0$$

$$\Leftrightarrow$$

$$x=-3 \quad \underline{\text{or}} \quad x=-1$$

3. Find the values of x where the equality is true:

$$\frac{(x+1)(x-2)}{(x+4)(x-6)} = 0$$

Remember:

$$\frac{a}{b}=0 \Leftrightarrow a=0$$

$$\Leftrightarrow$$

$$(x+1)(x-2)=0$$

$$\Leftrightarrow$$

$$x=-1 \quad \underline{\text{or}} \quad x=2$$

4. Find the values of x where the equality is true:

$$(x-1)\cdot\frac{x^2+5x+3}{x-1} = 1\cdot(x-1)$$

$$\Leftrightarrow$$

$$x^2+5x+3 = x-1$$

$$\Leftrightarrow$$

$$x^2+4x+4=0$$

$$\Leftrightarrow$$

$$(x+2)(x+2)=0$$

$$\boxed{\text{this is true} \Leftrightarrow x=-2}$$

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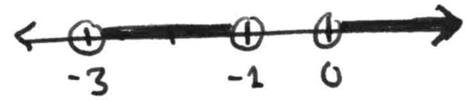
Solving Inequalties

1. Find the set of x where the inequality is true:

$\boxed{\text{true for } x \text{ in } (-3,-1)\cup(0,\infty)}$

$$x^3 + 4x^2 + 3x > 0$$



First: find when = holds

$x = 0, -3, -1$

Plot: with open circle since the inequality is FALSE at $x = 0, -3, -1$

$x(x^2+4x+3) > 0$

$x(x+3)(x+1) > 0$

CHECK each interval

$x = -4 \Rightarrow (-4)(-4+3)(-4+1) = (-4)(-1)(-3)$ ✗

$x = -2 \Rightarrow (-2)(-2+3)(-2+1) = (-2)(1)(-1)$ ✓

$x = -\frac{1}{2} \Rightarrow (-\frac{1}{2})(-\frac{1}{2}+3)(\ldots$

$= (-\frac{1}{2})(2.5)(.5)$ ✗

$x = 1 \Rightarrow (1)(1+3)(1+1)$

$= 1\cdot 4\cdot 2$ ✓

2. Find the set of x where the inequality is true:

$$x^2 + 5x + 6 \le 0$$

$(x+2)(x+3) \le 0$

First: = holds when

$x = -2$ or $x = -3$

Plot with closed circle since inequality is TRUE at $x = -2, -3$

CHECK each interval

$x = -4 \Rightarrow (-4+2)(-4+3) = (-2)(-1) = 2$ ✗

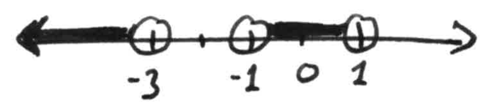
$x = -2.5 \Rightarrow (-2.5+2)(-2.5+3) = (-.5)(.5)$ ✓

$x = 0 \Rightarrow (0+2)(0+3) = 2\cdot 3$ ✗

$\Rightarrow$ true for x in $[-3,-2]$

3. Find the set of x where the inequality is true:

$$\frac{x+3}{(x+1)(x-1)} < 0$$



First: = holds when $x = -3$

(plot with open circle because of <)

Second: undefined when $x = 1$ or $x = -1$

(always plot with open circle)

CHECK each interval

$x = -4 \Rightarrow \frac{-4+3}{(-4+1)(-4-1)} = \frac{-1}{(-3)(-5)}$ ✓

$x = -2 \Rightarrow \frac{-2+3}{(-2+1)(-2-1)} = \frac{1}{(-1)(-3)}$ ✗

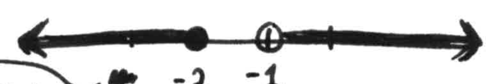
$x = 0 \Rightarrow \frac{0+3}{(0+1)(0-1)} = \frac{3}{(1)(-1)}$ ✓

$x = 2 \Rightarrow \frac{2+3}{(2+1)(2-1)} = \frac{5}{3\cdot 1}$ ✗

$\boxed{\text{true for } x \text{ in } (-\infty,-3)\cup(-1,1)}$

4. Find the set of x where the inequality is true:

$$\frac{(x+2)}{(x+1)} = \frac{(x+2)\cancel{(x+3)}}{(x+1)\cancel{(x+3)}} = \frac{x^2 + 5x + 6}{x^2 + 4x + 3} \ge 0$$



First: = holds when $x = -2$

(plot with closed circle because of $\ge$)

Second: undefined at $x = -1$

(plot w/ open circle)

Check: each interval

$x = -3 \Rightarrow \frac{(-3+2)}{(-3+1)} = \frac{(-1)}{(-2)}$ ✓

$x = -1.5 \Rightarrow \frac{(-1.5+2)}{(-1.5+1)} = \frac{.5}{-.5}$ ✗

$x = 0 \Rightarrow \frac{0+2}{0+1} = \frac{2}{1}$ ✓

$\boxed{\text{true for } x \text{ in } (-\infty,-2]\cup(-1,\infty)}$

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Functions

1. Define in words what it means for $f(x)$ to be a "function".

$f(x)$ is a function if each input gives ONE output

2. Let $f(x) = x^2 + x + 1$. Find $f(0)$, $f(1)$, and $f(2)$.

$$f(0) = (0)^2 + (0) + 1 = 1$$

$$f(1) = (1)^2 + (1) + 1 = 3$$

$$f(2) = (2)^2 + (2) + 1 = 4 + 2 + 1 = 7$$

3. Let $f(x) = x^2 + x + 1$. Write down and simplify $f(x+1)$ and $\dfrac{f(1+h) - f(1)}{h}$.

$$f(x+1) = (x+1)^2 + (x+1) + 1$$

$$= x^2 + 2x + 1 + x + 1 + 1$$

$$= x^2 + 3x + 3$$

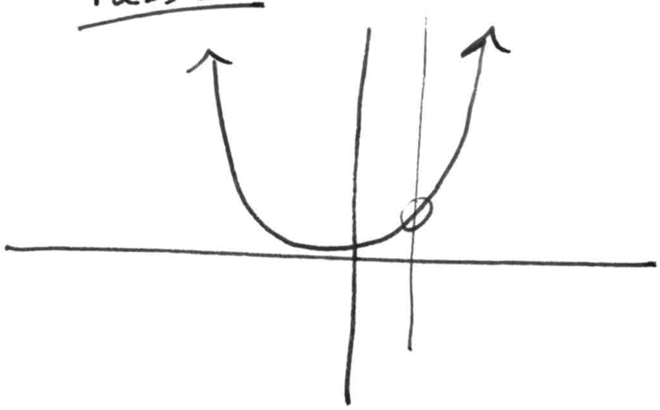
$$\frac{f(1+h) - f(1)}{h} = \frac{[(1+h)^2 + (1+h) + 1] - [1^2 + 1 + 1]}{h}$$

$$= \left[\frac{1 + 2h + h^2 + 1 + h + 1 - 3}{h}\right]$$

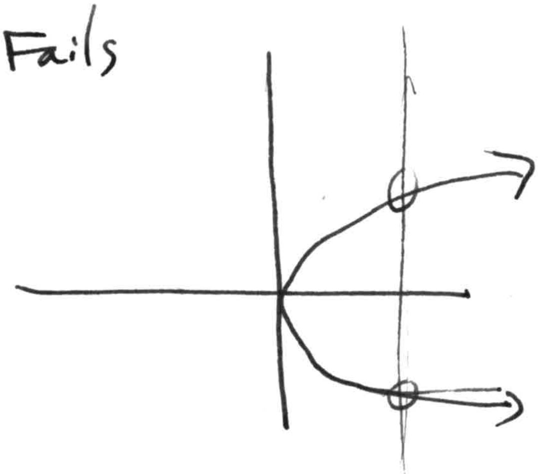
$$= \frac{h^2 + 3h}{h} = h + 3$$

4. Sketch one graph that passes the vertical line test, and one that fails the vertical line test.

Passes:



Fails



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5. Define the "domain" of a function $f(x)$ in words.

the domain of f
is the set of numbers x
where $f(x)$ is defined

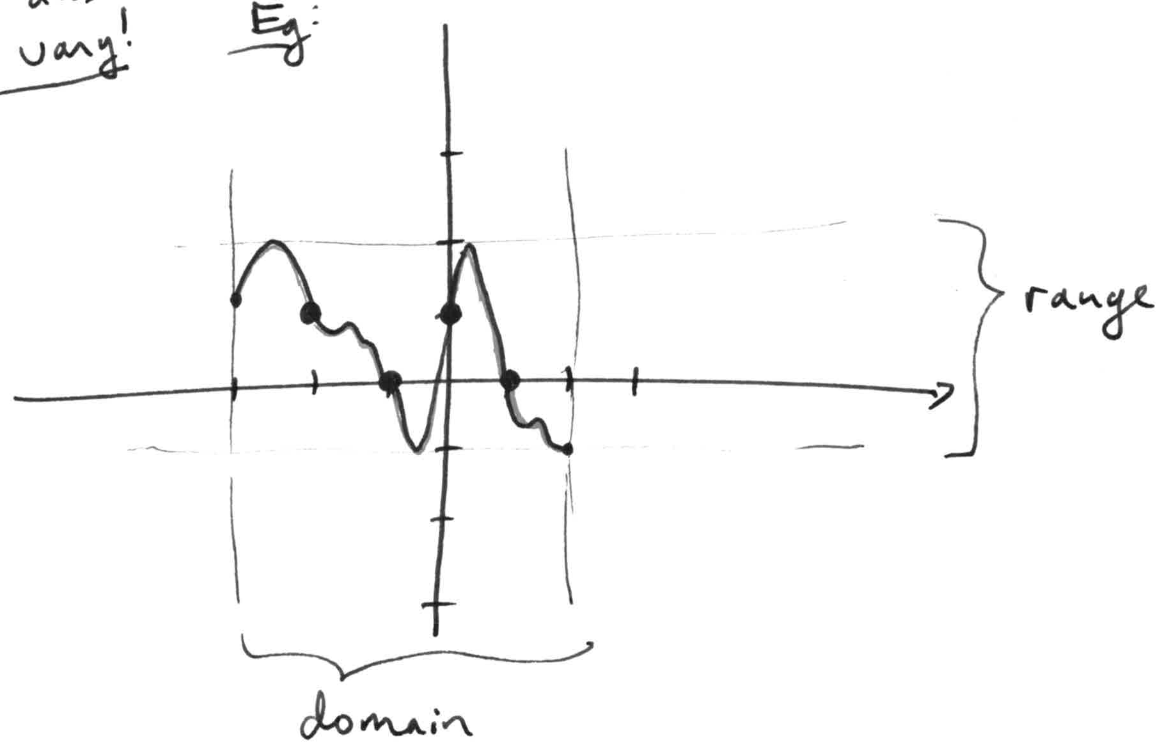
6. Define the "range" of a function in words.

the range of f
is the set of numbers y
output by f
(that is: the y s.t. $f(x)=y$ for some # x)

7. Sketch the graph of a function with domain $[-3,2]$ and range $[-1,2]$ such that $f(-2)=1$, $f(-1)=0$, $f(0)=1$, and $f(1)=0$.

your answer may vary!

Eg:



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Finding Domains Algebraically

1. Find the domain of $f(x) = \sqrt{x}$

$\sqrt{x}$ is defined $\Leftrightarrow$ $x \geq 0$

Domain $= [0, \infty)$

2. Find the domain of $f(x) = \sqrt{x+1}$

$\sqrt{x+1}$ is defined $\Leftrightarrow$ $x+1 \geq 0$

$\Leftrightarrow$ $x \geq -1$

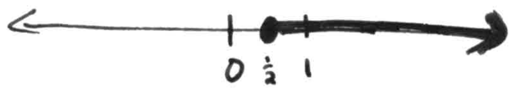
Domain $= [-1, \infty)$

3. Find the domain of $f(x) = \sqrt{2x-1}$

$\sqrt{2x-1}$ is defined $\Leftrightarrow$ $2x-1 \geq 0$

$\Leftrightarrow$ $2x \geq 1$

$\Leftrightarrow$ $x \geq \frac{1}{2}$



Domain $= [\frac{1}{2}, \infty)$

4. Find the domain of $f(x) = \dfrac{x+1}{(x-1)(x+7)}$

$f(x)$ is defined $\Leftrightarrow$ $(x-1)(x+1) \neq 0$

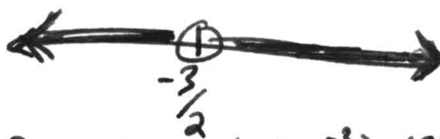
$\Leftrightarrow$ $x \neq 1$ AND $x \neq -1$

Domain $= (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

5. Find the domain of $f(x) = \dfrac{1}{(2x+3)^2}$

$f(x)$ is defined $\Leftrightarrow$ $(2x+3)^2 \neq 0$

$\Leftrightarrow$ $2x+3 \neq 0$ $\Leftrightarrow$ $\boxed{x \neq \frac{-3}{2}}$



Domain $= (-\infty, \frac{-3}{2}) \cup (\frac{-3}{2}, \infty)$

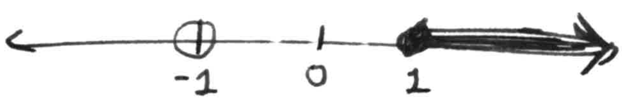
6. Find the domain of $f(x) = \dfrac{\sqrt{x-1}}{x+1}$

$f(x)$ is defined $\Leftrightarrow$ BOTH $x - 1 \geq 0$

AND $x \neq -1$

$\Leftrightarrow$ BOTH $x \geq 1$

AND $x \neq -1$



Domain $= [1, \infty)$

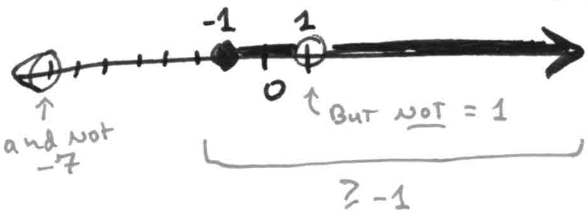
7. Find the domain of $f(x) = \dfrac{\sqrt{x+1}}{(x-1)(x+7)}$

$f(x)$ is defined $\Leftrightarrow$ BOTH $x + 1 \geq 0$

AND $x \neq 1$

AND $x \neq -7$

$\Leftrightarrow$ $\boxed{\text{BOTH } x \geq -1 \text{ AND } x \neq 1 \text{ AND } x \neq -7}$



Domain $= [-1, 1) \cup (1, \infty)$

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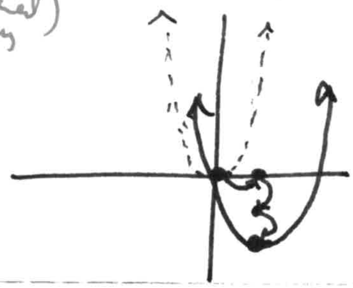
Sketching Quadratics

1. Sketch the graph and find the vertex of $f(x) = 2(x-1)^2 - 2$

① start with $2x^2$ (x^2 stretched vertically)

② move right 1

③ move down 2



vertex is $(1,-2)$

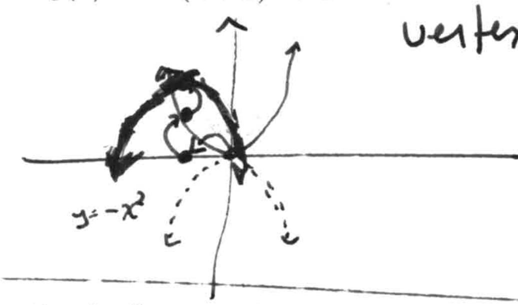
2. Sketch the graph and find the vertex of $g(x) = -1(x+1)^2 + 2$

① start with x^2

② reflect accross x-axis

③ move left 1

④ move up 2



vertex is $(-1,2)$

3. Sketch the graph of the quadratic function by first completing the square.

$$h(x) = x^2 + 6x - 8$$

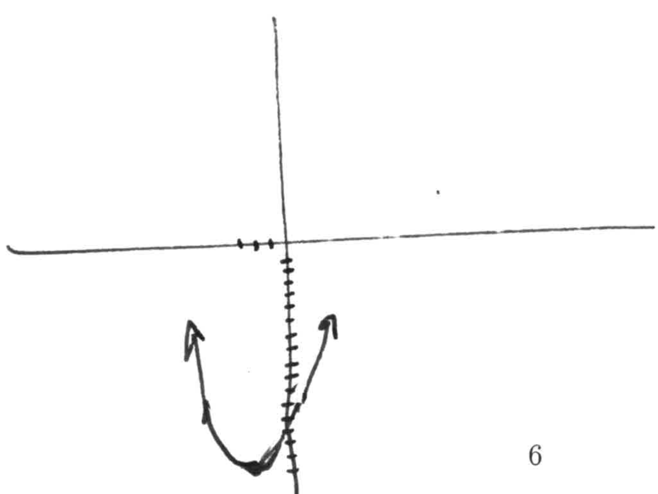
$$= x^2 + 6x + \underline{9} - \underline{9} - 8$$

$$= (x+3)(x+3) - 9 - 8$$

$$h(x) = (x+3)^2 - 17$$

so $h(x)$ is x^2 moved ① left 3

② down 17



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Sketching Polynomials

$\boxed{\text{leading term is } x^3}$

1. Sketch the graph of the polynomial $f(x) = x(x+1)(x-2)$

(1) End behavior: leading term is x^3

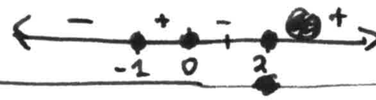
$\Rightarrow$ 

(2) $f(x) = 0$

$\Leftrightarrow$ $x(x+1)(x-2) = 0$

$\Leftrightarrow$ $x = 0$ or $x = -1$ or $x = 2$

(3) check when positive/negative



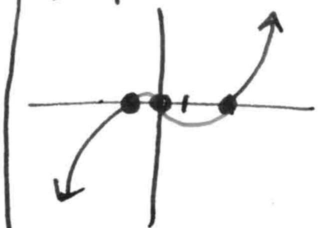
$x = -2 \Rightarrow f(-2) = \cdots = (-2)(-1)(-4)$ ⊖

$x = -\frac{1}{2} \Rightarrow f(-\frac{1}{2}) = \cdots = (\frac{-1}{2})(\frac{1}{2})(\frac{-5}{2})$ ⊕

$x = 1 \Rightarrow f(1) = \cdots = (1)(2)(-1)$ ⊖

$x = 3 \Rightarrow f(3) = \cdots = (3)(4)(1)$ ⊕

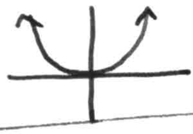
Graph:



$\boxed{\text{leading term is } x^4}$

2. Sketch the graph of the polynomial $g(x) = x(x+1)(x-2)(x+3)$

① End Behavior: leading term is x^4

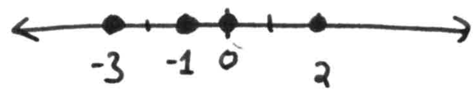
$\Rightarrow$ 

② $f(x) = 0$

$\Leftrightarrow$ $x(x+1)(x-2)(x+3) = 0$

$\Leftrightarrow$ $x = 0$ or $x = -1$ or $x = 2$ or $x = -3$

(3) check where positive / negative



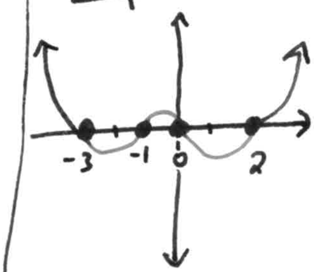
$x = -4 \Rightarrow f(-4) = (-4)(-3)(-6)(-1)$ ⊕

$x = -2 \Rightarrow f(-2) = (-2)(-1)(-4)(1)$ ⊖

$x = \frac{-1}{2} \Rightarrow f(\frac{-1}{2}) = (\frac{-1}{2})(\frac{1}{2})(-2.5)(2.5)$ ⊕

$x = 3 \Rightarrow f(3) = 3 \cdot 4 \cdot 1 \cdot 6$ ⊕

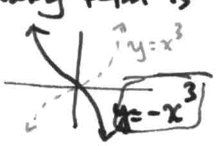
Graph:



3. Sketch the graph of the polynomial $h(x) = -x^2(x-1)$ $\leftarrow$ ($\boxed{\text{leading term is } -x^3}$)

① End behavior

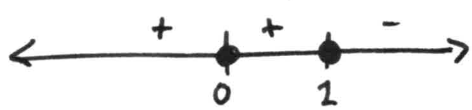
leading term is $-x^3$

$\Rightarrow$ 

② $f(x) = 0$

$\Leftrightarrow$ $-x^2(x-1) = 0 \Leftrightarrow$ $x = 0$ or $x = 1$

(3) Check where positive/negative

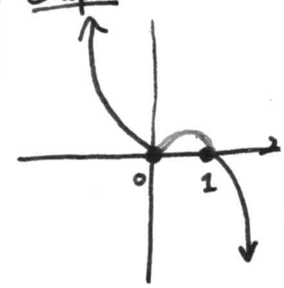


$x = -1 \Rightarrow f(-1) = -(-1)^2(-1-1) = -(1)(-2)$ ⊕

$x = \frac{1}{2} \Rightarrow f(\frac{1}{2}) = -(\frac{1}{2})^2(\frac{1}{2}-1) = -(\frac{1}{2})(\frac{-1}{2})$ ⊕

$x = 2 \Rightarrow f(2) = -(2)^2(2-1) = -4(1)$ ⊖

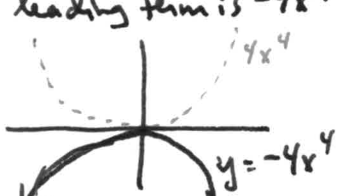
Graph:



4. Sketch the graph of the polynomial $p(x) = -4(x+1)^2(x-2)^2$ $\leftarrow$ $\boxed{\text{leading term is } -4x^4}$

① End Behavior

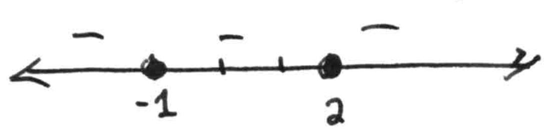
leading term is $-4x^4$



② $f(x) = 0$

$\Leftrightarrow$ $-4(x+1)^2(x-2)^2 = 0 \Leftrightarrow$ $x = -1$ or $x = 2$

(3) Check where positive/negative



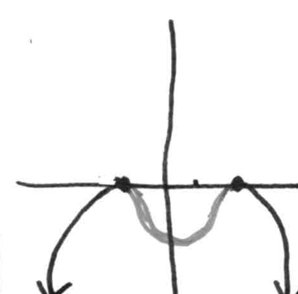
$x = -2 \Rightarrow f(-2) = -4(-2+1)^2(-2-2)^2$

$= -4(-1)^2(-4)^2$ ⊖

$x = 0 \Rightarrow f(0) = -4(0+1)^2(0-2)^2$ ⊖

$x = 3 \Rightarrow f(3) = -4(3+1)^2(3-2)^2$ ⊖

Graph



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Sketching Rational Functions

1. Sketch the graph of the rational function

$$f(x) = \frac{x+1}{(x-1)(x+2)}$$

2. Sketch the graph of the rational function

$$g(x) = \frac{x^2+1}{x-1}$$

3. Sketch the graph of the rational function

$$h(x) = \frac{2x+3}{7x-4}$$